ASSIGNMENT 2
REPORT

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Optimal Power Flow
with Electricity and Heat Stores

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1 Introduction

In this report I attempt to model electricity network with electricity and heat storage over multiple periods. First, I write down the mathematical model and then try to solve that model using Mosel XPress. After building model in mosel, I will try to use that approach to solve for the three different cases:

- with CO_2 tax and no PAR
- with CO_2 tax and PAR on one of the lines
- with CO_2 tax and one extra line along one of the original lines

The idea here is to explore the effect of carbon tax on generation of electrical power. Then, to find how PAR and extra lines can affect the generation and distribution of energy along lines.

2 Mathematical Model Formulation

2.1 Base model

During the model formulation phase, I have followed the approach presented in Topics 3.3 and 4. Since, I have used the "DC approximation" to model the power flow, I had to assume the following:

- line resistances to be negligible compared to the line reactances in order to simplify the parameters;
- voltage magnitudes to be normalized to 1;
- voltage angle differences between the neighbouring buses to be "small" (taking advantage of the fact that the $\sin x = x$ and $\cos x = 1$ for small angle x).

Similarly to the example Ex3-L123 that was part of the previous assignment, I wish to calculate optimal power flow in the system. I want to minimize the cost of power generation and here, I need to consider the carbon tax, hence taking into consideration the CO_2 impact the energy production has on the environment.

First, I started by declaring sets of indices. This is important as the following parameters and variables will be indexed using the notation presented here.

Set	Description
G	set of generators, indexed $g = 1, \ldots, \lvert G\rvert$
B	set of buses, indexed $b = 1, \ldots, \lvert B\rvert$
L	set of lines, indexed $l = 1, \ldots, \lvert L\rvert$
T	set of periods, indexed $t = 1, \ldots, \lvert T\rvert$
B'	set of buses to which heat and electricity stores are connected
F	set of fuels, indexed $f \in \{water, coal, gas\}$
S	set of stores, indexed $s \in \{S_1, H_1\}$
S^H	subset of set S of heat stores
S^E	subset of set S of electric stores

In the table below I present all the parameters used for the model. This is the information (data) given in the exercise.

Parameter	Description
H_t	length of period t
tax	carbon tax in $
P_{bt}^{D}	power demand of load at bus b in period t
P_{l}^{L+}	maximum power flow in line l
P_{g}^{G+}	maximum generation from generator g
a_{bl}	element of bus/line, -1 if b is the start bus of l, 1 if b is the end point of l, 0 otherwise
C_{g}^{G}	cost of power generation from generator g
β_g	bus to which generator g is connected
X_l	reactance of line l
E_{g}^{G}	efficiency of generator g
W_f	emissions from generator f
P_{s}^{S+}	maximum power into store s
P_{s}^{S-}	maximum power out of store s
E_{s}^{S}	store's s efficiency
H^S	number of hours of full output from electricity store when full
Q^{mass}	increase in energy stored in heat store per unit rise of temperature
P^{loss}	heat power loss from heat store per unit temperature difference between interior and exterior
T_{t}^{S+}	maximum allowed temperature at heat store at start of period t
T_{t}^{S-}	minimum allowed temperature at heat store at start of period t
T_{t}^{ext}	exterior temperature of the heat store at time t
PAR^{max}	maximum positive phase angle difference allowed by PAR
PAR^{min}	maximum negative phase angle difference allowed by PAR
PAR^{pos}	line to which PAR is connected

Finally, I present the decision variables used within the model.

Variable	Description
p_{lt}^{L}	power flow into line l at period t from its start bus
p_{gt}^{G}	power generated by generator g at period t
δ_{b}^{B}	voltage phase angle at bus b
p_{st}^{S}	power into store s at period t (used to generate heat at start of period t at heat store)
q_{st}^{S}	level of store s at start of period t (heat energy at heat store)
PAR_l	value of the phase angle regulator at line l

The point of the exercise is to minimize the operating cost while ensuring all the constraints hold. Here the operating cost consists of: the cost of power generation from each of generators and the carbon tax multiplied by tonnes of CO_2 emissions that is a byproduct of energy generation process. I start formulating model with the objective function is given by:

$$\min \sum_{g\in G}\sum_{t\in T} C_g^G \cdot H_t \cdot p_{gt}^G + \sum_{g\in G}\sum_{t\in T} tax \cdot H_t \cdot p_{gt}^G \cdot \frac{W_f}{E_g^G}$$

This objective function is subject to the following constraints:

$$\sum_{g\in G|\beta_g=b} p_{gt}^G + \sum_{l\in L} a_{bl} p_{lt}^L = P_{bt}^D \quad \text{for all} \quad b \in B \setminus B', t \in T \tag{1}$$

$$\sum_{g\in G|\beta_g=b} p_{gt}^G + \sum_{l\in L} a_{bl} p_{lt}^L + \sum_{s\in S} p_{st}^S = P_{bt}^D \quad \text{for all} \quad b \in B', t \in T \tag{2}$$

$$X_l p_{lt}^L + \sum_{b\in B} a_{bl}\delta_b^B = 0 \quad \text{for all} \quad l \in L, t \in T \tag{3}$$

$$-P_l^{P+} \le p_{lt}^L \le P_l^{P+} \quad \text{for all} \quad l \in L, t \in T \tag{4}$$

$$-P_g^{G+} \le p_{gt}^G \le P_g^{G+} \quad \text{for all} \quad g \in G, t \in T \tag{5}$$

$$P_s^{S-} \le p_{st}^S \le P_s^{S+} \quad \text{for all} \quad s \in S, t \in T \tag{6}$$

$$q_{st+1}^S = \begin{cases} q_{st}^S + E_s^S p_{st}^S H_t, & \text{for} \quad p_{st}^S \ge 0 \\ q_{st}^S + p_{st}^S H_t, & \text{for} \quad p_{st}^S < 0 \end{cases} \quad \text{for all} \quad t \in T, s \in S^E \tag{7}$$

$$q_{st}^S \le H^S P_s^{S+} \quad \text{for all} \quad t \in T, s \in S^E \tag{8}$$

$$T_t^{S-} \le \frac{q_{ht}^S}{Q^{mass}} \le T_t^{S+} \quad \text{for all} \quad t \in T, h \in S^H \tag{9}$$

$$q_{ht+1}^S = \gamma_{ht} q_{ht}^S + g(H_t) - \gamma_{ht} g(0) \quad \text{for all} \quad t \in T, h \in S^H \tag{10}$$

$$\gamma_{ht} = \exp\{-\lambda_h H_t\} \quad \text{for all} \quad t \in T, h \in S^H \tag{11}$$

$$g(0) = \left(\frac{a_{ht}}{\lambda_h} - \frac{b_{ht}}{\lambda_h^2}\right) \quad \text{for all} \quad t \in T, h \in S^H \tag{12}$$

$$g(H_t) = \left(\frac{a_{ht}}{\lambda_h} - \frac{b_{ht}}{\lambda_h^2}\right) + \frac{b_{ht}}{\lambda_h} H_t \quad \text{for all} \quad t \in T, h \in S^H \tag{13}$$

$$\lambda_h = \frac{P^{loss}}{Q^{mass}} \tag{14}$$

$$a_{ht} = E_h^S p_{ht}^S + P^{loss} T_t^{ext} \quad \text{for all} \quad t \in T, h \in S^H \tag{15}$$

$$b_{ht} = \left(E_h^S \left(p_{ht+1}^S - p_{ht}^S\right) + P^{loss}\left(T_{t+1}^{ext} - T_t^{ext}\right)\right)\frac{1}{H_t} \quad \text{for all} \quad t \in T, h \in S^H \tag{16}$$

Where (1) and (2) are Kirchhoff's Current Law constraints, (3) is Kirchhoff's Voltage Law, (4) line and (5) generation constraints. (6) ensures that power into store stays between its limits. (7) ensures that level at store is equal to the level in previous period plus (minus) power that went into the store multiplied by the store's efficiency (went out of the store). (8) is the upper bound of the store's level. (9) makes sure that the interior temperature lies between the upper and lower bounds allowed. Constraint (10) models energy level at the heat store in the next period. Finally, constraints (11)-(16) allow to model constraint (10) and the change in heat energy at heat store between next and current period.

2.2 Adding PAR to the model

PAR in the line allows for the phase difference between the end of the line and its attached bus to occur. In the exercise, we consider PAR to be in a range of ± 0.5 radians. This can be used to control the flow in the lines withing our system. For the purpose of model formulation, PAR affect only the KVL constraint, which becomes:

$$X_l p_{lt}^L + \sum_{b \in B} a_{bl} \delta_b^B + PAR_l = 0 \quad \text{for all} \quad l \in L, t \in T$$

where $PAR_l \in \mathbb{R}^L$ is a vector which takes a value of PAR when PAR is placed on line l and zeros otherwise.

3 Scenarios

Having implemented the above model in Mosel Xpress, I was able to solve for the three scenarios given in the assignment. Here the operating cost consists of: the cost of power generation from each of generators and the carbon tax multiplied by tonnes of CO_2 emissions that is a byproduct of energy generation process. In other words, we take into consideration the external cost imposed on others by energy generation. We will further check that when interpreting the results.

3.1 Carbon tax with no PAR

Run the Mosel Xpress file without making any changes.

Characteristic	Value
Average hourly cost	61924.62
Average hourly cost excluding CO2 tax	46840.54
Average hourly CO2 emissions	301.68
Average power output from G1	0.155
Average power output from G2	0.089
Average power output from G3	0.229
Average power input to electricity store	0.178
Average power input to heat store	0.060
Difference between maximum and minimum energy in the electricity store	0.6
Maximum temperature in heat store	22

3.2 Carbon tax with PAR

One needs to comment out the KVL constraint in line 109. And uncomment the "model with PAR" part of the Mosel file (lines 155-164).

Characteristic	Value
Average hourly cost	57396.10
Average hourly cost excluding CO2 tax	35222.16
Average hourly CO2 emissions	443.48
Average power output from G1	0.155
Average power output from G2	0.238
Average power output from G3	0.082
Average power input to electricity store	0.147
Average power input to heat store	0.085
Difference between maximum and minimum energy in the electricity store	0.6
Maximum temperature in heat store	22

3.3 Doubling the line

For doubling the line, one needs to alter the data file. In order to add a line, one needs to add a column in to a_{bl} in the data file. On top of that, one needs to add a line "L4" in the `setL: ["L1" "L2" "L3"]`, add reactance to `Xl: [2 1 0.5]` and finally the line capacity to `PlLplus: [0.1 0.55 0.15]`. Therefore, when doubling the line l_1, the data needs to be of the following format:

- `setL: ["L1" "L2" "L3" "L4"]`
- `Xl: [2 1 0.5 2]`
- `PlLplus: [0.1 0.55 0.15 0.1]`
- `abl: [-1 0 -1 -1 1 -1 0 0 0 1 1 1]`

Characteristic	Value
Average hourly cost	55403.67
Average hourly cost excluding CO2 tax	43738.46
Average hourly CO2 emissions	233.304
Average power output from G1	0.1998
Average power output from G2	0.049
Average power output from G3	0.224
Average power input to electricity store	0.167
Average power input to heat store	0.059
Difference between maximum and minimum energy in the electricity store	0.6
Maximum temperature in heat store	22

And similarly, for other lines.

For duplicating L2 we need to change the data file:

- `setL: ["L1" "L2" "L3" "L4"]`
- `Xl: [2 1 0.5 1]`
- `PlLplus: [0.1 0.55 0.15 0.55]`
- `abl: [-1 0 -1 0 1 -1 0 -1 0 1 1 1]`

Characteristic	Value
Average hourly cost	63969.83
Average hourly cost excluding CO2 tax	51685.08
Average hourly CO2 emissions	245.69
Average power output from G1	0.154
Average power output from G2	0.029
Average power output from G3	0.288
Average power input to electricity store	0.149
Average power input to heat store	0.059
Difference between maximum and minimum energy in the electricity store	0.6
Maximum temperature in heat store	22

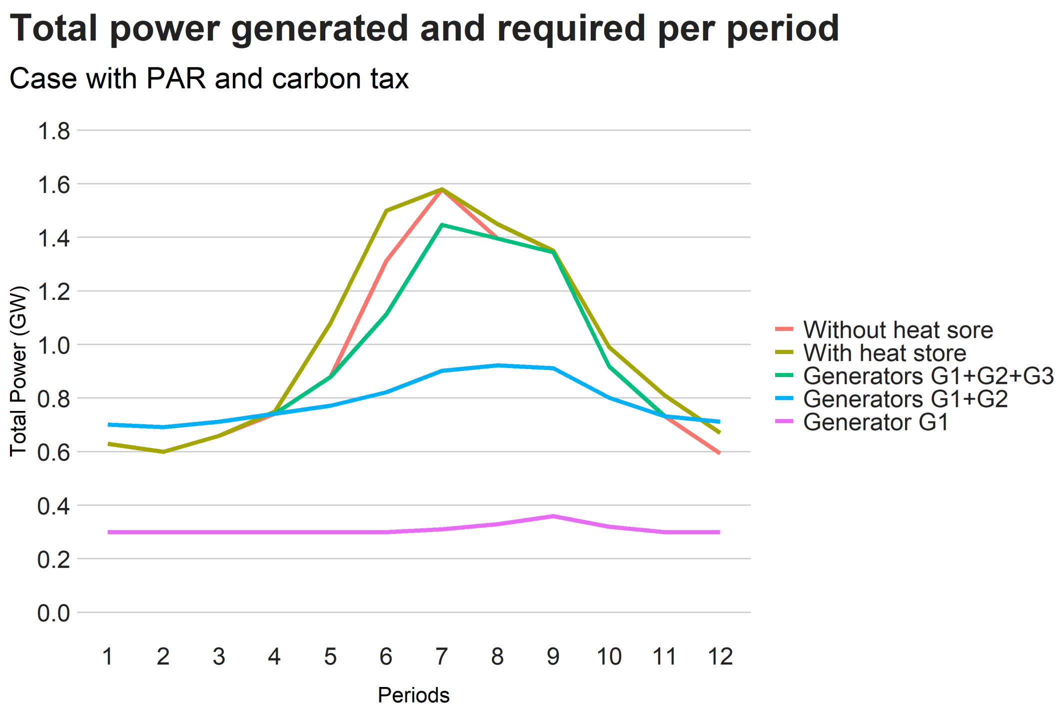
For duplicating L3 we need to change the data file:

- `setL: ["L1" "L2" "L3" "L4"]`
- `Xl: [2 1 0.5 0.5]`
- `PlLplus: [0.1 0.55 0.15 0.15]`
- `abl: [-1 0 -1 -1 1 -1 0 0 0 1 1 1]`

Characteristic	Value
Average hourly cost	51191.73
Average hourly cost excluding CO2 tax	33372.69
Average hourly CO2 emissions	356.380
Average power output from G1	0.2
Average power output from G2	0.179
Average power output from G3	0.093
Average power input to electricity store	0.178
Average power input to heat store	0.071
Difference between maximum and minimum energy in the electricity store	0.6
Maximum temperature in heat store	22

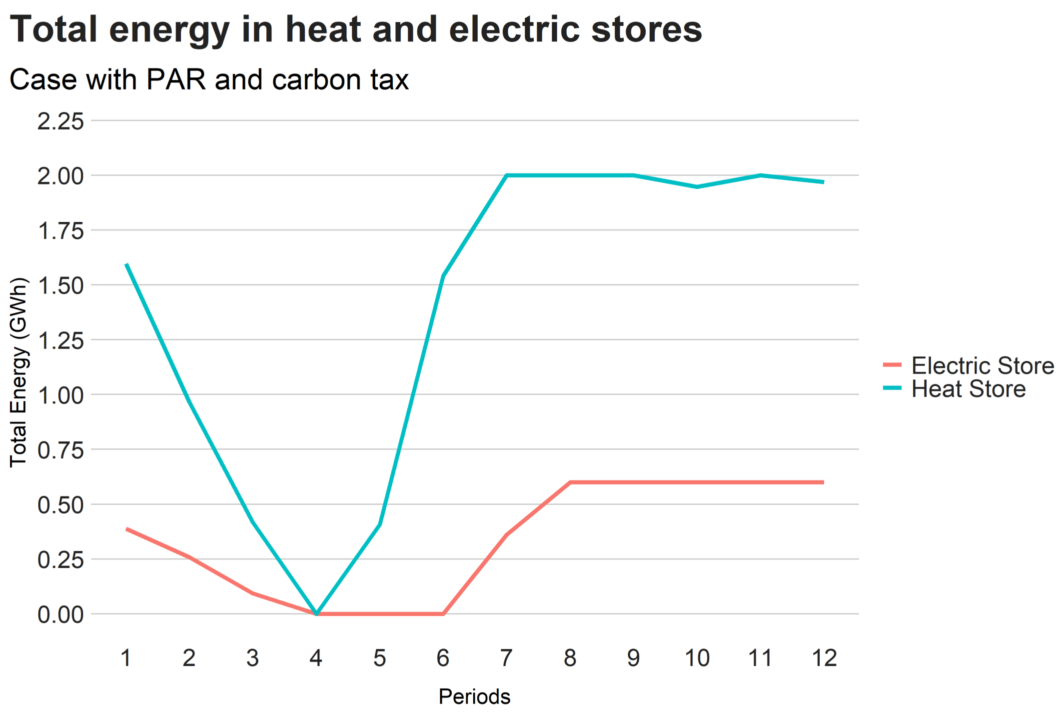
4 Plots

a)



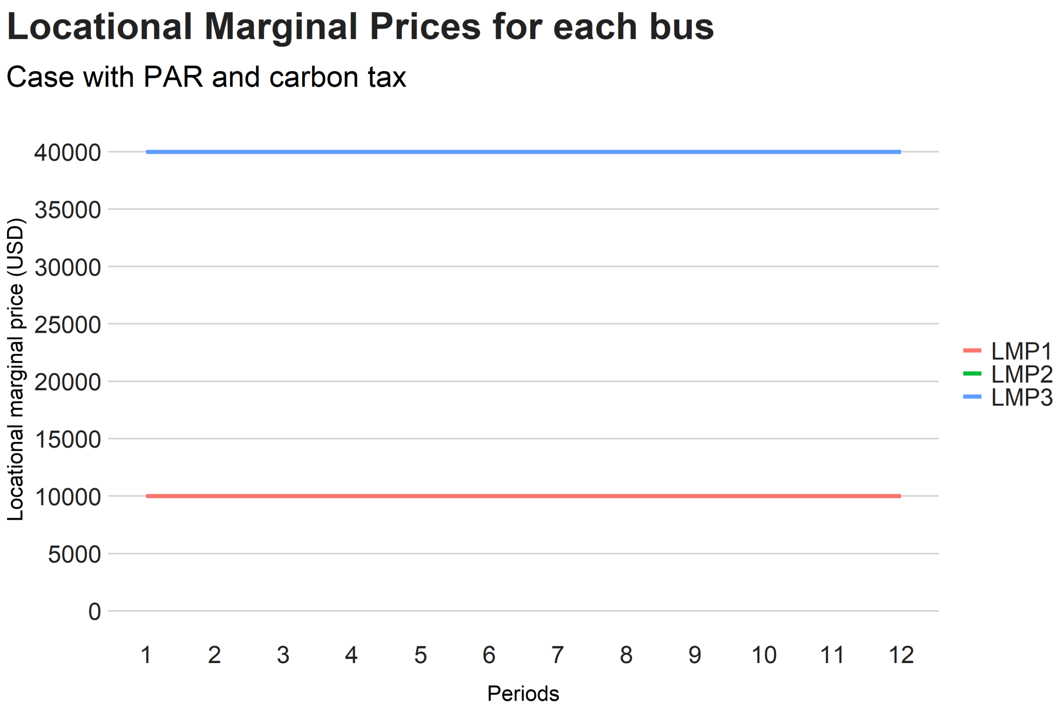
Total power generated and required per period
Case with PAR and carbon tax
Total Power (GW)
1.8
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
1 2 3 4 5 6 7 8 9 10 11 12
Periods
Without heat sore
With heat store
Generators G1+G2+G3
Generators G1+G2
Generator G1

b)



Total energy in heat and electric stores
Case with PAR and carbon tax
Total Energy (GWh)
2.25
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.00
1 2 3 4 5 6 7 8 9 10 11 12
Periods
Electric Store
Heat Store

c)



Locational marginal prices are constant across periods. In addition, LMP2 = LMP3. The reason for that can be the line capacity for lines l_1 and l_3. I have checked what happens if the line capacity changes. When increased, the power flow changes and locational marginal prices can differ across periods.

5 Questions

a) Explain whether or not when installing a duplicate line which has the same reactance as the existing line there is an advantage in having a higher capacity in the new line than in the existing line.

In electrical network power is driven by differences in voltages. Ohm's Law states that the voltage drop across a resistor is equal to the current flowing through this resistor times the resistance (reactance) of this resistor. Kirchhoff's Voltage Law is analogous to Ohm's Law for the case of DC circuits. In the DC approximation, the flow p^L_{lt} depends on the voltages on the ends of the buses. Hence, here the power flow at time t in line l in terms of voltage phase angles is given by:

$$p^L_{lt} = -\frac{V^2}{X_l} \sum_{b \in B} a_{bl} \delta_{bt}$$

where V is assumed to be constant and equal to $V = 1$. Therefore, if I was to duplicate line with the same reactance as the existing line there would be no advantage in having a higher capacity in the new line than in the existing line. I would have to ensure that both lines have higher line capacities in order to observe any advantages.

I have also checked that experimentally using the Mosel Xpress model prepared for this assignment. When additional line was installed (with the same reactance as in the already existing line) the power flow in the original and duplicate lines is the same. When I have allowed for a larger line capacity in the duplicate line, there was no change in the power flow. On the other hand, when I have decreased the line capacity below the level of the original one, the power flow decreased in both lines. This stays in line with what I have written above about the KVL and power flow.

b) In Task 2(b) is it possible to install a PAR with a smaller maximum angle without increasing the cost, and if so what is the smallest angle maximum angle.

Yes, it is possible to install a PAR with a smaller maximum angle without increasing the cost. With the PAR given in the exercise with maximum angle difference of ± 0.5 radians, the actual angle taken by a PAR is ± 0.2969. This means that it would be possible to install a PAR with a maximum angle difference of ± 0.3 radians (or even a bit smaller) without increasing the cost. The smallest maximum angle would be ± 0.2969 according to my model.

c) In Task 2(c) with L1 duplicated, find what the minimum possible CO2 output is and with that minimum output what the operating cost is.

With line L1 duplicated, I have run the model in order to minimize the total CO_2 emissions. In the Mosel Xpress I have changed the `minimize(total_cost)` to `minimize(total_pollution)`. The output I got was: 5314.979 tonnes of CO_2 per day or 221.457 tonnes of CO_2 per hour. The corresponding operational cost is $\$1339471.903$ per day or $\$55811.329$ per hour. When compared to the scenario where I wanted to minimize the total cost, the total CO_2 emissions were 5599.304 tonnes per day or 233.304 tonnes of CO_2 per hour. In the base scenario, the operating cost was equal to $\$1329688.159$ per day or $\$55403.673$ per hour. This means that the minimum possible CO_2 output is not being achieved when minimizing the operational costs in a scenario with a duplicated line L1.