# TAOR REPORT 2021

## FIFA World Cup 2022 Lodging Capacity Planning Project

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## **Executive Summary**

In 2022 Qatar will host the FIFA World Cup where 32 nations compete in a group phase and a knock-out phase. Total spectator attendance is heaviest in the group stage when multiple matches are held daily. In previous years around half a million foreign spectators needed lodging in the host cities. The problem is that Qatar does not have this lodging capacity, so this must be built in time for the World Cup. A construction project of this magnitude needs adequate planning. This paper determines lodging requirements for the next World Cup with a three-part model. The first part is called the Group Formation Model. This is an integer programming problem which aims to partition the 32 nations in the group stage into 8 groups of equal strength based on the nations' respective FIFA points (the world ranking points) system). This is done as per the competition's constraints on group formation. The second part is called the Group-Letter Assignment Model. This is another integer programming model which assigns matches to stadiums based on a pre-determined schedule. It attempts to predominantly assign popular matches to the high capacity stadiums to ensure maximal attendance. The third part predicts the total required lodging based on the schedule from the second model by predicting the foreign attendance to the matches and the duration of the foreign spectators' stay in Qatar. Bringing everything together, we arrive at an estimate for the maximum foreign attendance of 66,880 on a given day which can be used to plan accommodation provision. We carry out a brief sensitivity analysis to take into account the as yet unknown affect that the ongoing pandemic will have on the attendance protocols at this tournament, concluding a linear relationship between stadium capacity reduction and a reduced need for accommodation. We then finish by discussing the strengths and limitations of the methodology, noting that there is likely to be a costly over-estimation of required lodging by virtue of the "worst-case scenario" approach. We also explain how the "spectator index" is a potentially flawed metric, with a number of stark assumptions that haven't been fully justified.

## 1 Introduction

Every four years 32 teams from six confederations compete in the FIFA World Cup. These confederations include: the Asian (AFC), African (CAF), North/Central American and Caribbean (CONCACAF), South American (CONMEBOL) and European (UEFA) Football Confederations. We disregard the Oceanian Football Confederation (OFC) as those nations have relatively low chances of qualifying based on their previous performance. The tournament consists of a group stage in which 48 matches are played and a knock-out phase in which 16 matches are played. Historically, these tournaments are attended by about three million spectators of which half a million need lodging in the host country. The tournaments are usually hosted by UEFA or CONMEBOL countries with large tourism sectors, however, in 2022 the event will be hosted in Qatar which has very little tourism infrastructure. The aim of this report is to develop a methodology for estimating lodging requirement based on the work in "Prescriptive analytics for FIFA World Cup lodging capacity planning" by Ghoniem et al. [1]. This paper will henceforth be referred to as "the paper". The paper was written by researchers of the University of Massachusetts, the University of Qatar and the Qatar Tourism Authority with funding from the Qatar Government. Their overall method consists of two integer programming models and a further calculation-based model to determine the lodging requirement. The model only looks at the group stage as this is when lodging requirements are highest due to the amount of foreign spectators in Qatar.

## 2 Methodology

## 2.1 Framework for attendance analytics

Match attendance and therefore overall lodging requirements are based on the quality and popularity of nations that qualify and the capacity of the stadiums in which they play. In our report we assume that countries with the highest FIFA ranking as of February 2021 (respecting the required proportion of teams involved from each confederation) are the ones that will qualify, plus the host nation which

qualifies by default. We also use a "spectator index" to quantify the propensity of a nation's fans to attend their games. This is a weighted average of the ratio of expected attendance (calculated based on South Africa 2010 attendance) and offered seating capacity for each of their three group stage games. In Appendix A there is a table of all 32 nations that we assumed will qualify for the tournament and their respective FIFA points, taken from [2], along with their spectator indices which have either been taken directly from [1] or calculated using their methodology. Note that the method for calculating FIFA points has changed since the paper was written, which may yield interesting results.

## 2.2 Group Formation Model

Our first task is to sort the teams into groups to determine who will play who in the group stage of the competition. The constraints are that we can only have one team from each confederation per group - with the exception of UEFA, who can have two teams per group, and that there is only one team from each pot in each group. Teams are sorted into pots by way of their FIFA points total - in other words, every team in pot 1 has more FIFA points than every team in pot 2 and so on. We aim to maximise the total number of FIFA points in each group. Ordinarily, teams are sorted randomly into groups (subject to the constraints) but optimising in this way gives a worst-case scenario in terms of accommodation provision as more competitive games will likely attract the largest number of spectators. The mathematical formulation of this model can be seen in Appendix B. We have slightly reformulated the objective function from what is found in [1] to something which is equivalent but more easily interpretable. Running the model in Xpress we arrive at the group allocation in Table 1. Note that the headings of "Team 1", "Team 2" etc. and the order in which teams appear in the table is arbitrary. The important factor is the groupings.

## 2.3 Group-Letter Assignment Model

The purpose of Group-Letter Model is to spread matches predicted to be most popular across different stadiums while ensuring that more popular matches are

Group	Team 1	Team 2	Team 3	Team 4
1	Australia	Senegal	Columbia	England
2	Algeria	Mexico	Argentina	Poland
3	Korea	Uruguay	Wales	Spain
4	Tunisia	Jamaica	France	Italy
5	Nigeria	Chile	Netherlands	Portugal
6	Japan	Morocco	Brazil	Switzerland
7	Iran	Costa Rica	Belgium	Germany
8	Qatar	USA	Croatia	Denmark

Table 1. Optimised groupings

assigned to stadiums with higher capacity. Following the paper's methodology, we introduce the match popularity index. We assume that the match attendance includes supporters from two playing parties, fans from other nations and officials. For a match between two nations  $i_1$  and  $i_2$  in the same group, the match popularity is defined as the sum of spectator indices for: the first nation  $f_{i_1} \in [0,1]$ , the second nation  $f_{i_2} \in [0,1]$ , other nations  $\hat{f}_{(i_1,i_2)} = \frac{f_{i_1}+f_{i_2}}{2}$  and officials  $\tilde{f}_{(i_1,i_2)} = 1$  if  $\max\{f_{i_1}, f_{i_2}\} = 1$ 1 and  $\tilde{f}_{(i_1,i_2)} = \frac{f_{i_1}+f_{i_2}}{2}$  otherwise. We then introduce the notion of row-set popularity which is the sum of match popularity for matches scheduled in the row, that is, each row represents a different stadium in our typical group stage-schedule which we introduce later. The model ensures that row-sets with higher popularities are allocated to higher capacity stadiums. We are constrained by the fact that each game in a given group must take place at a unique stadium and that the host nation must play the opening match at the flagship stadium (usually the national stadium). The mathematical formulation can be seen in Appendix C. Running the model in Xpress we arrived at an optimal group lettering. The obtained solution is then used to assign row-sets to the stadiums of different capacities based on the pre-determined "typical" tournament schedule outlined in [1]. The table in Appendix D gives an outline of this schedule with the resultant matches filled in. It also details the capacity of the assigned stadiums, the corresponding row-set popularity, and the popularity of each of the 48 matches, ranging from 0.24 to 4.00.

#### 2.4 Foreign spectator attendance and lodging requirements

The final model brings everything together in producing a concrete prediction for how much lodging is required. We must estimate how many foreign individuals are likely to attend each game, and from that work out how many beds will be needed each night. We first introduce two seat allocation parameters:  $\alpha$  is the proportion of seats reserved for officials and  $\beta$  is the proportion of the remaining seats offered to each of the two nations involved and to other nations. For the purposes of our calculation we will use  $\alpha = 0.09$  and  $\beta = 0.12$  as per [1]. The formulae for expected attendances for each foreign group at any match can be found in Appendix E - these form the basis for our calculation. Now that we know how many people will likely attend each game, we need to take into account how long each of these people are likely to stay in Qatar. We will assume that supporters either attend one of each of the three groups stage games, the first two, the last two or all three. We ignore the unlikely case where they attend the first and third but not the second. We also assume that supporters arrive in Qatar on the day of their first game and leave the day after their last. Their length of stay is roughly determined by proximity to Qatar and GDP per capita, more precisely, we denote the probability of attending all three matches for a neighbouring nation by  $p_{123}^N$  and two matches by  $p_{12}^N$  and  $p_{23}^N$ . The same is done for non-neighbouring countries but replacing the superscript N with H for high GDP and L for low GDP. We used a GDP per capita of \$25,000 as the cut-off between high and low. These probabilities can be seen in the table in Appendix F. We can then use these probabilities, along with our expected attendances to compute the number of spectators attending each permutation of matches. Note that this only needs to be done for the supporters of each of the two competing nations as we assume that everyone else attends one match on average. The exact formulae can be found in Appendix G. Finally, we multiply each of these values by the number of days spanning the first and last matches they attend to get the total number of nights. This information then needs to be manually rationalised to form the total number of foreign attendees for each night. The calculation spreadsheets can be seen in Appendix H.



## Total foreign visitors per night

## Figure 1. Graph of visitors per night

We can arrive at the conclusion, based on the graph above, that we need to provide accommodation for a maximum of 66,880 visitors.

## 3 Sensitivity analysis

The paper's authors have performed the analysis on 16 permutations of 32 nations that are likely to qualify. They conclude that increment increases in the spectator index or the probabilities of extended stay are likely to increase the lodging capacity needed for the World Cup even more than they have estimated. Additionally, we believe that Covid 19 might have an effect on stadium capacities and influence the lodging capacity needed. Due to local restrictions there may be a significant reduction in available stadium capacity. Changing stadiums' capacity does not affect our estimates in Group Formation nor in Group-Letter Assignment models. The group stage schedule that we would obtain is going to remain the same, only the available capacity of each stadium will decrease by some percentage. If Qatar was to limit the number of arrivals of foreign supporters, it would be possible to occupy the stadiums with local spectators – however this would not affect the lodging

capacity needed. Therefore, when decreasing the stadiums' capacity by k%, the maximum lodging capacity will also fall by k% other things constant, according to our methodology.

To our contention, because of the influence of Covid-19, Qatar should be wary of overestimating required lodging capacity. Currently in 2021, Covid-19 has a large impact in events of this scale. For example, Japan is not allowing any foreign spectators at the Tokyo Olympics [3]. While the necessity for such measures is improbable for an event in (late) 2022 due to global Covid-19 vaccination programs, it is not impossible. As Covid-19 is caused by a rapidly mutating virus, a variant might arise which greatly reduces the efficacy of the vaccines and as many countries are lacking behind with their vaccination programs, there is more time for such a variant to arise [4]. Such a variant could force Qatar to take action, reducing the amount of foreign spectators still need to maintain distance from one another which could prevent stadiums from utilising their full capacity. To sum up, we think it is risky to assume that Covid-19 will have no impact on the event and that Qatar should take this into account when estimating required lodging capacity.

## 4 Discussion

We believe a strength of the paper's methodology is the fact that it calculates a range of expected lodging requirements based on 16 different scenarios that could unfold. The competing nations will not be known until a few months in advance of the tournament and the construction of the lodging capacity has to start many years in advance to allow for construction lead time. Based on this range the paper concludes that the minimum requirement of FIFA is too low for all 16 scenarios. However, the model is designed to gauge maximal attendance and determines lodging requirements in a "worst-case scenario". In practise, the groups are determined semi-randomly. This means that it is likely that the actual number of people in need of lodging will be smaller and thus sufficient lodging will be provided in any case.

in building surplus accommodation.

To estimate match popularity they propose a "spectator index", which estimates the percentage of seats allocated to a qualifying nation that will be filled. We question the validity of the method used to calculate this. Firstly, the spectator index is partly based on spectator attendance of the World Cup in South Africa in 2010, which was not the most recent World Cup at the time of writing. In an article written by the University of Massachusetts it is stated that the author plans to incorporate the data of the 2014 tournament in Brazil into the model [5]. Secondly, there is little justification of the method that is used to calculate the spectator index and it leads to odd results. According to the paper, the spectator index is lower for countries that are distant from the host nation and higher for countries with high GDPs and loyal fanbases. According to the paper, England, France and Germany have spectator indices of 100, 51 and 63 respectively. These countries have similar geographic locations and GDPs, so it seems to us there is more justification needed to explain this disparity. By using this methodology, the spectator indices are surprisingly low for football-loving countries like Croatia. This in turn affects the match popularity and therefore, we obtain matches like: Denmark vs. Croatia with the match popularity of only 0.24 out of a possible 4. Additionally, it is written in the paper that neighbouring countries are given an index of 100%, but there is no justification provided based on data from previous tournaments that justifies this assumption (none of South Africa's neighbours qualified for the World Cup anyway). On top of that, Egypt, which is not a direct neighbour of Qatar, was also given an index of 100% in their case study. It is not clear what qualifies as a "neighbouring" country".

We think the spectator indices are based on strong assumptions without sufficient justification. The quality of the model is therefore highly dependent on the quality of the assumptions and estimations of the writers. The spectator index needs to be better justified and possibly tuned before it is applicable to use in the model.

#### 5 Conclusions and Recommendations

The paper was written to assess the preparedness of Qatar's lodging capacity in anticipation of the Qatar World Cup 2022. The paper describes a three part model, which optimises team groupings, fits these to a match schedule and produces an estimate of foreign spectator attendance. The paper analyses 16 different scenarios of 32 qualifying nations and three levels for the spectator index and probabilities of extended stay of foreign spectators. This results in 67,000 required rooms averaged over the scenarios, which is 7000 above the minimum of 60,000 as recommended by FIFA. This requirement increases as spectator index and probability of extend stay increase. In our model we updated the qualifying teams and considered the teams who are most likely to qualify. Following the paper's methodology we arrive at a maximum of 66,880 rooms required. This is similar to the paper's estimate. We believe that the paper is overestimating the lodging requirement at great cost. In the first part of the model the group formations are optimised to create groups with close to equal strength. The second model assigns the most popular matches to the largest stadiums. This creates a "worst-case scenario" in terms of lodging capacity needed. In reality, groups are formed semi-randomly and it is improbable that this exact scenario will unfold. Secondly, we have identified multiple ways Covid-19 could impact the capacities of the stadiums. We think it is a risky assumption that Covid-19 will not impact the event at all (in terms of foreign attendance). Therefore, we think Qatar should take into account the probability that stadiums will not be able to run at full capacity or that certain nations may not be able to attend the event. Lastly, we think the spectator indices which are assigned to all nations are determined by a possibly invalid methodology. This methodology contains old data, badly justified assumptions and estimates by the author. This will contribute to general inaccuracy of the models. We think that reviewing this methodology and by taking into account the randomness of the group assignments and the impact of Covid-19, a more accurate estimate can be determined to assess the preparedness of Qatar's lodging capacity.

## References

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## Appendices

## Appendix A

Nation	Confederation	FIFA points	Spectator Index(%)
Qatar	AFC	1391	100
Australia	AFC	1457	77
Iran	AFC	1496	100
Japan	AFC	1502	29
Korea	AFC	1465	13
Algeria	CAF	1488	50
Senegal	CAF	1558	16
Morocco	CAF	1474	35
Nigeria	CAF	1474	27
Tunisia	CAF	1503	50
CostaRica	CONCACAF	1427	3
Jamaica	CONCACAF	1437	1
Mexico	CONCACAF	1632	63
USA	CONCACAF	1545	100
Argentina	CONMEBOL	1642	54
Brazil	CONMEBOL	1743	94
Chile	CONMEBOL	1567	26
Colombia	CONMEBOL	1601	17
Uruguay	CONMEBOL	1639	9
Belgium	UEFA	1780	38
Croatia	UEFA	1755	6
Denmark	UEFA	1614	6
England	UEFA	1670	100
France	UEFA	1755	51
Germany	UEFA	1610	63
Italy	UEFA	1625	51
Netherlands	UEFA	1609	55
Portugal	UEFA	1662	34
Switzerland	UEFA	1593	13
Wales	UEFA	1562	4
Spain	UEFA	1645	51
Poland	UEFA	1559	10

Table 2. 32 Nations assumed to qualify

## Appendix B

## Notation:

- N: Set of 32 qualifying nations.
- $p_i$ : FIFA points for nation  $i, \forall i \in N$ .
- C: Set of confederations.
- $c_{ij}$ :  $c_{ij} = 1$  if and only if nation *i* belongs to confederation *j*,  $c_{ij} = 0$  otherwise.
- $\kappa_j$ : Max teams from confederation j per group.
- $P_k \subset N, k = 1,...,4$ : Pot of eight nations at the  $k^{th}$  level of FIFA points;  $p_i \ge p_j, \forall i \in P_{k_1}, j \in P_{k_2} | k_1 < k_2$ .
- G: Index set of groups to be formed.
- *x<sub>ig</sub>*: Binary variable, 1 if and only if nation
   *i* is assigned to group *g*, 0 otherwise.
- w: Objective value.

The objective function along with constraint (1) ensures minimum total FIFA points in a group is maximised, creating groups of similar strengths. Constraint (2) ensures each group contains four teams whilst constraint (3) ensures each team is only assigned to one group. Constraint (4) means no two teams from the same pot can be in the same group and constraint (5) enforces the confederation constraints. Constraint (6) is simply a non-negativity condition.

## Optimization problem:

 $\max w$  s.t.

$$w \le \sum_{i \in N} p_i x_{ig}, \quad \forall g \in G \tag{1}$$

$$\sum_{i \in N} x_{ig} = 4, \quad \forall g \in G \tag{2}$$

$$\sum_{g \in G} x_{ig} = 1, \quad \forall r \in R \tag{3}$$

$$x_{i_1g} + x_{i_2g} \le 1, \quad \forall g \in G; i_1, i_2 \in P_k$$
 (4)

$$\sum_{i \in N} x_{ig} c_{ij} \le \kappa_j, \quad \forall g \in G, j \in C$$
(5)

$$w \ge 0 \tag{6}$$

## Appendix C

## Notation:

- R: index set for row-sets
- *G*: index set for eight groups obtained in the Group Formation Model
- L: set of letters {A,B,C,D,E,F,G,H}, host nation should belong to Group A
- $M_g$ : set of matches for subset  $g, \forall g \in G$ , ordered based on FIFA ranking
- $\pi_{gm}$ : popularity of match m in subset g,  $\forall g \in G \ m \in M_g$
- $I_r$ : set of matches in row-set  $r \in R$
- $z_{gl}$ : binary variable, 1 if the subset g is assigned to group letter l, 0 otherwise
- $y_r$ : row-set popularity for  $r \in R$
- $w^{\min}$ : minimum row-set popularity
- $w^{max}$ : maximum row-set popularity
- $v_r$ : binary variable to affect computation of the maximum row-set popularity

Constraint (1) enforces maximization of the minimum row-set popularity while (2) enforces maximization of the maximum row-set popularity. Constraints (3) and (4) ensure that the objective is bounded. Constraint (5) computes the row-set popularity. Constraints (6) and (7) are assignment constraints for the eight team subsets and group letters. Constraint (8) pre-assigns the group A to the host nation's subset. Finally, (9) is the non-negativity constraint.

## Optimization problem:

 $\max w^{min} + w^{max}$  s.t.

$$w^{min} \le y_r, \quad \forall r \in R$$
 (7)

$$w^{max} \ge y_r, \quad \forall r \in R$$
 (8)

$$w^{max} \le 16(1-v_r), \quad \forall r \in R$$
 (9)

$$\sum_{r \in R} v_r = 1 \tag{10}$$

$$y_r = \sum_{g \in G} \sum_{l \in L} \sum_{\substack{m \in M_g: \\ (l,m) \in I_r}} \pi_{gm} z_{gl}, \quad \forall r \in R$$
 (11)

$$\sum_{g \in G} z_{gl}, \quad \forall l \in L$$
 (12)

$$\sum_{l\in L} z_{gl}, \quad \forall g \in G \quad (\mathbf{13})$$

$$z_{8A} = 1$$
 (14)

$$y_r, w^{min}, w^{max} \ge 0 \tag{15}$$

## Appendix D

Row	Stadium	Capacity								Day								Row-set
			1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	popularity
1			QAT							ARG				IRN			JAM	
			CRO							POL				GER			FRA	
	Lusail	86250	2.59							1.28				3.445			1.04	8.355
2				USA			NGA			AUS					ALG			
				DEN			CHL			COL					ARG			
	AlRayyan	44740		2.59			1.06			1.88					2.08			7.61
3				BEL			NED				JAP					URY		
				GER			POR				BRA					WAL		
	AlKhor	45330		2.02			1.78				2.46					0.26		6.52
4				IRN				TUN				KOR			AUS			
				CRC				JAM				URY			ENG			
_	AlShamal	45120		2.545				1.02				0.44			3.655			7.66
5					COL			FRA				WAL			MEX			
		( 5100			ENG			IIA				SPN			POL			3366
,	AlWakrah	45120			2./55			2.04				1.1		000	1.46			7.355
0	E.I				AUS						ALG			CRO			NGA	
	Education	( 5750			SEIN 10/						MEX 2.2/			DEN			POR	F F0
7	City	45350			1.80 MEV			OAT			2.20			0.24	CEN		I.ZZ	5.58
/								DEN							SEIN			
	AlGharafa	44740			23/			2 50				12			0.66			6 70
0	AIGHUIUIU	44740			2.54			2.37	1104			1.2			0.00	MDC		0.79
0									CPO				NED			RDA		
	UmmSlal	45120			12				2 50				164			2.58		8.01
0	ommolai	45120			1.2	RRA			2.57	SEN			1.04	OAT		2.50	СНІ	0.01
						SW7				FNG				USA			NFD	
	Khalifa	68030				2 14				2 74				4			162	10.5
10	ra la la	00000				JAP			CRC	2.7 1			.JAM			KOR		10.0
						MRC			GER				ITA			SPN		
	DohaPort	44950				1.28			1.32				1.04			1.28		4.92
11						URG			IRN				TUN			JAP		
	Sports					SPN			BEL				FRA			SWZ		
	City	47560				1.2			3.07				2.02			0.84		7.13
12	-						KOR				MRC			CRC			TUN	
	Qatar						WAL				SWZ			BEL			ITA	
	University	43520					0.34				0.96			0.82			2.02	4.14

#### Table 3. Group stage schedule

## Appendix E

- Officials:  $\tilde{F}_m = \tilde{f}_{(i_1,i_2)} \alpha \kappa_{(i_1,i_2)};$
- Nation 2:  $F_m^{i_2} = f_{i_2}(1-\alpha)\beta\kappa_{(i_1,i_2)};$
- Nation 1:  $F_m^{i_1} = f_{i_1}(1-\alpha)\beta\kappa_{(i_1,i_2)};$
- Others:  $\hat{F}_m = \hat{f}_{(i_1,i_2)}(1-\alpha)\beta\kappa_{(i_1,i_2)}$ .

## Appendix F

Nei	ghbo	ur	Lov	w GD	Ρ	Hi	igh GE	)P
$p_{123}^N$	$p_{12}^{N}$	$p_{23}^{N}$	$p_{123}^L$	$p_{12}^{L}$	$p_{23}^{L}$	$p_{123}^{H}$	$p_{12}^{H}$	$p_{23}^{H}$
0.05	0.1	0.1	0.05	0.1	0.1	0.1	0.15	0.15

## Appendix G

- All three:  $\Phi_{123}^i = p_{123} \min\{F_{m_1}^i, F_{m_2}^i, F_{m_3}^i\}$ ; One only:  $\Phi_1^i = F_{m_1}^i \Phi_{123}^i \Phi_{12}^i$ ;
- Two and three:  $\Phi_{23}^i = p_{23} \min\{F_{m_2}^i, F_{m_3}^i\}$ ; Two only:  $\Phi_2^i = F_{m_2}^i \Phi_{123}^i \Phi_{12}^i \Phi_{23}^i$ ;
- One and two:  $\Phi_{12}^i = p_{12} \min\{F_{m_1}^i, F_{m_2}^i\}$ ; Three only:  $\Phi_3^i = F_{m_3}^i \Phi_{123}^i \Phi_{23}^i$ .

## Appendix H

Nation	Game 1	Game 2	Game 3	Total	Game day 1	Game day 2	Game day 3	P12	P23	P123	All three	2 and 3	1 and 2	1 only	2 only	3 only
AUS	3813	3762	3794	11369	3	8	13	0.15	0.15	0.1	376	564	564	2873	2257	2853
SEN	792	1189	782	2763	3	8	13	0.05	0.1	0.1	78	78	40	675	993	625
COL	838	831	831	2499	3	8	13	0.05	0.1	0.1	83	83	42	713	623	664
ENG	4927	7429	4927	17283	3	8	13	0.15	0.15	0.1	493	739	739	3695	5458	3695
ALG	2464	2476	2443	7382	3	9	13	0.05	0.1	0.1	244	244	123	2096	1864	1954
MEX	3078	3120	3104	9302	3	9	13	0.05	0.1	0.1	308	310	154	2616	2348	2486
ARG	2638	5086	2638	10362	3	8	13	0.05	0.1	0.1	264	264	132	2242	4426	2111
POL	493	942	493	1927	3	8	13	0.05	0.1	0.1	49	49	25	419	819	394
KOR	618	641	638	1896	5	10	14	0.15	0.15	0.1	62	96	93	463	390	481
URU	467	443	446	1356	4	10	14	0.05	0.1	0.1	44	44	22	401	333	357
WAL	190	197	198	585	5	10	14	0.15	0.15	0.1	19	30	29	143	120	149
SPN	2649	2513	2503	7665	4	10	14	0.15	0.15	0.1	250	376	377	2021	1510	1878
TUN	2464	2597	2376	7437	6	11	15	0.05	0.1	0.1	238	238	123	2103	1998	1901
JAM	49	49	94	193	6	11	15	0.05	0.1	0.1	5	5	2	42	37	84
FRA	2513	2649	4803	9965	6	11	15	0.15	0.15	0.1	251	397	377	1885	1623	4155
ITA	2513	2503	2424	7440	6	11	15	0.15	0.15	0.1	242	364	376	1895	1522	1818
NGA	1319	1330	1337	3987	5	11	15	0.05	0.1	0.1	132	133	66	1121	999	1072
CHI	1270	1270	1932	4472	5	10	15	0.05	0.1	0.1	127	127	64	1080	953	1677
NED	2723	2710	4086	9518	5	11	15	0.15	0.15	0.1	271	406	406	2045	1626	3408
POR	1683	1661	1684	5028	5	10	15	0.15	0.15	0.1	166	249	249	1268	997	1268
JAP	1423	1436	1506	4365	4	9	14	0.15	0.15	0.1	142	215	214	1068	864	1148
MOR	1718	1663	1724	5106	4	9	14	0.05	0.1	0.1	166	166	83	1468	1248	1392
BRA	6983	4653	4631	16268	4	9	14	0.05	0.1	0.1	463	463	233	6287	3494	3705
SWZ	966	618	675	2259	4	9	14	0.15	0.15	0.1	62	93	93	811	371	521
IRN	4927	5194	9419	19539	2	7	12	0.05	0.1	0.1	493	519	246	4188	3935	8406
CRC	148	147	143	438	2	7	12	0.05	0.1	0.1	14	14	7	126	111	114
BEL	1881	1974	1806	5660	2	7	12	0.15	0.15	0.1	181	271	282	1418	1240	1354
GER	3119	3092	5934	12145	2	7	12	0.15	0.15	0.1	309	464	464	2345	1855	5161
USA	4886	4927	7429	17242	2	7	12	0.15	0.15	0.1	489	739	733	3664	2967	6201
CRO	565	296	297	1158	1	7	12	0.05	0.1	0.1	30	30	15	521	222	238
DEN	293	293	297	883	2	6	12	0.15	0.15	0.1	29	44	44	220	176	224

Table /	Attandanca	calculation	cummany
10DIE 4.	Allendunce	calculation	summary

		0	-		E	1	-	0	0	40		40	47		45
Day	1	2	3	4	5	6	/	8	9	10	11	12	13	14	15
AUS	0	0	3813	940	940	940	940	3761	940	940	940	940	3793	0	0
SEN	0	0	793	118	118	118	118	1189	156	156	156	156	781	0	0
COL	0	0	838	125	125	125	125	831	166	166	166	166	830	0	0
ENG	0	0	4927	1232	1232	1232	1232	7429	1232	1232	1232	1232	4927	0	0
ALG	0	0	2463	367	367	367	367	367	2475	488	488	488	2442	0	0
MEX	0	0	3056	440	440	440	440	440	3098	596	596	596	3082	0	0
ARG	0	0	2638	396	396	396	396	5086	528	528	528	528	2639	0	0
POL	0	0	493	74	74	74	74	942	98	98	98	98	492	0	0
KOR	0	0	0	0	618	155	155	155	155	641	158	158	158	639	0
URU	0	0	0	467	66	66	66	66	66	443	88	88	88	445	0
WAL	0	0	0	0	191	48	48	48	48	198	49	49	49	198	0
SPN	0	0	0	2648	627	627	627	627	627	2513	626	626	626	2504	0
TUN	0	0	0	0	0	2464	361	361	361	361	2597	476	476	476	2377
JAM	0	0	0	0	0	49	7	7	7	7	49	10	10	10	94
FRA	0	0	0	0	0	2513	628	628	628	628	2648	648	648	648	4803
ITA	0	0	0	0	0	2513	618	618	618	618	2504	606	606	606	2424
NGA	0	0	0	0	1319	198	198	198	198	198	1330	265	265	265	1337
CHI	0	0	0	0	1271	191	191	191	191	1271	254	254	254	254	1931
NED	0	0	0	0	2722	677	677	677	677	677	2709	677	677	677	4085
POR	0	0	0	0	1683	415	415	415	415	1661	415	415	415	415	1683
JAP	0	0	0	1424	356	356	356	356	1435	357	357	357	357	1505	0
MOR	0	0	0	1717	249	249	249	249	1663	332	332	332	332	1724	0
BRA	0	0	0	6983	696	696	696	696	4653	926	926	926	926	4631	0
SWZ	0	0	0	966	155	155	155	155	619	155	155	155	155	676	0
IRN	0	4927	739	739	739	739	5193	1012	1012	1012	1012	9418	0	0	0
CRC	0	147	21	21	21	21	146	28	28	28	28	142	0	0	0
BEL	0	1881	463	463	463	463	1974	452	452	452	452	1806	0	0	0
GER	0	3118	773	773	773	773	3092	773	773	773	773	5934	0	0	0
USA	0	4886	1222	1222	1222	1222	4928	1228	1228	1228	1228	7429	0	0	0
CRO	566	45	45	45	45	45	297	60	60	60	60	298	0	0	0
DEN	0	293	73	73	73	293	73	73	73	73	73	297	0	0	0
Officials	7763	10148	9533	5855	3215	7134	9676	10500	5755	2772	4879	14934	8301	5078	7721
Other	4992	7626	9521	7103	3902	6358	7815	9619	6983	3363	5919	16376	9505	6162	9368
TOTAL	13321	33071	41411	34191	24098	32112	42333	49237	37418	24951	33825	66880	42834	26913	35823

Note in Table 5 that the greyed out squares are days on which a match falls for the country concerned. This made it easier to visualise when each countries attendance needed to be updated as the calculation was done by hand.